Idiosyncratic Kurtosis and Expected Returns

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While many asset pricing models assume normality in the return distribution, most evidence suggests that this assumption is often violated (Fama [1965] and Rosenberg [1974]). The presence of large price run-ups and subsequent reversals in securities markets is a reminder that stocks tend to exhibit excess kurtosis. Indeed, extreme positive and/or negative returns happen more regularly than predicted by a normal distribution. For example, the stock market crash of October 1987 is an occurrence that, according to a normal distribution, would happen only once every 150 million years. Events that would be extremely rare under a normal distribution seem to occur empirically 10 times more often than they otherwise should (Idzorek and Xiong [2012]). These empirical facts are captured by kurtosis, which measures how the tails of a distribution differ from normality. If kurtosis represents some sort of risk that is unaccounted for in other more common risk measures, then extreme returns, on either side of the distribution, might influence asset prices. Furthermore, investment theory is founded on the idea of mean–variance efficiency, and excess kurtosis in the return distribution might reduce the accuracy of asset pricing models.

This study attempts to determine whether kurtosis discourages investor demand and, therefore, influences contemporaneous prices and expected returns. On the one hand, investors may perceive that stocks with excess kurtosis contain a different type of risk that is unaccounted for in traditional measures of volatility. To the extent that this is true, investors may only be willing to purchase these stocks at a discount. In this case, excess kurtosis may be associated with higher expected returns. This idea seems to be the most intuitive. An increased probability of ending up farther away from the mean because of kurtosis is similar to a distribution with a larger standard deviation. On the other hand, findings in Ang et al. [2006, 2009] and Frazzini and Pedersen [2014] suggest that various measures of risk are negatively associated with expected returns. A negative relation between risk and return is puzzling, and if kurtosis indeed represents some sort of risk, then, like these other studies, kurtosis may be negatively associated with expected returns.

To determine whether kurtosis affects asset prices in the cross section, we first sort stocks into portfolios based on a measure of firm-specific kurtosis. To obtain this measure, we first estimate a capital asset pricing model in order to obtain residual returns. We then estimate kurtosis, or the fourth scaled moment of the distribution, using the daily residual returns for a rolling six-month window. This measure is denoted as "idiosyncratic kurtosis" and captures the firm-specific thickness of the tails of the return distribution. We find that stocks that have the highest idiosyncratic kurtosis underperform stocks with the lowest idiosyncratic kurtosis. These results hold when we examine alphas from risk-factor models such as the capital asset pricing model as well as other common multifactor models.

To determine the cross-sectional return premium associated with idiosyncratic kurtosis, we use a standard technique originally used in Fama and MacBeth [1973] and examine the relation between next-month returns and idiosyncratic kurtosis. After controlling for additional factors that influence returns, we find a reliably negative relationship between idiosyncratic kurtosis and next-month returns. This negative association is robust to, among other things, controls for idiosyncratic volatility and idiosyncratic skewness. These findings support the results from our portfolio analysis and contribute to the knowledge about how investors view the risk associated with return distributions. More importantly, from a practical point of view, our findings suggest that, to the extent that kurtosis represents additional risk unaccounted for in traditional measures of volatility, investors do not appear to be compensated for this risk. If anything, investors seem to be overpaying for stocks with higher kurtosis.

DATA DESCRIPTION

The data used in this analysis come from two primary sources. From Compustat, we gather annual balance sheet information including assets, liabilities, and total equity. From the universe of stocks available from the Center for Research on Security Prices (CRSP), we obtain trading volume, prices, returns, shares outstanding, and so on and aggregate this data to the monthly level. From Wharton Research Data Services (WRDS), we gather the following risk factors at the monthly levels: MRP is the market risk premium or the CRSP value-weighted return less than the one-year U.S. T-bill yield; SMB is the Fama and French [1996] small-minus-big risk factor; HML is the Fama and French [1996] high-minus-low risk factor; and UMD is the Carhart [1997] up-minus-down risk factor.

We use a standard, daily market model in order to calculate several different variables. *Beta* is the slope coefficient from the market model. *IdioVolt* is the idiosyncratic volatility or the standard deviation of residual returns that are obtained from the market model. We also estimate higher-order moments of the return distribution using daily returns and residual returns from the market model.

$$Kurt\ or\ IdioKurt = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\left[\sum_{i=1}^{n}\frac{(r_t-\overline{r})^4}{\sigma^4}\right] - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{1}$$

First, we estimate two measures of kurtosis. *Kurt* is the kurtosis of the return distribution during a particular month, where returns are daily raw returns. *IdioKurt* is the idiosyncratic kurtosis and is estimated using Equation (1), where *r* is the residual return from the daily market model. We focus the tests in this article on idiosyncratic kurtosis, although we have replicated the analysis using *Kurt* and find very similar results.[1] We focus on idiosyncratic kurtosis so that our findings can be more easily compared with prior research, which predominantly uses idiosyncratic kurtosis. We also calculate monthly idiosyncratic skewness (*IdioSkew*), which is the skewness of daily residual returns using the six-month rolling window calculated using Equation (2).

$$IdioSkew = \frac{n}{(n-1)(n-2)}\left[\sum_{i=1}^{n}\frac{(r_t-\overline{r})^3}{\sigma^3}\right] \tag{2}$$

We obtain a variety of other variables that will be used as controls throughout the analysis. From CRSP, we calculate monthly market capitalization (*Size*) and Amihud's [2002] measure of illiquidity (*Illiq*), which is the ratio of the absolute value of daily returns to trading volume (in 100,000s). *Illiq* is calculated at the daily level and then averaged to the monthly level.

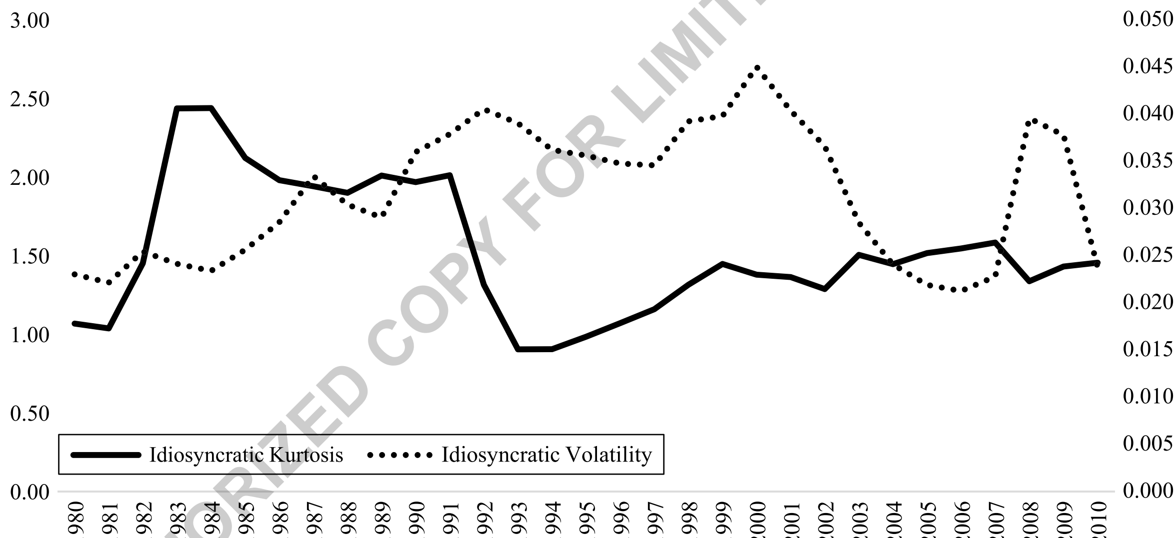
After gathering and calculating all of these variables for the intersection of stocks available from both CRSP and Compustat, we provide some restrictions to the data. First, to be included in our final sample, we require stocks to have a positive book-to-market ratio. Second, we exclude data prior to 1980 such that our final sample time period ranges from 1980 to 2010. Our final sample includes 19,267 unique stocks and 1,871,407 stock-month observations.

EXHIBIT 1
Summary Statistics

	Kurt	*IdioKurt*	*IdioSkew*	*IdioVolt*	*Beta*	*Size*	*B/M*	*Illiq*
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Mean	1.5968	1.5016	0.2301	0.0328	0.6615	1.6059	0.4061	1.0188
Median	0.6207	0.5734	0.2031	0.0244	0.6041	0.1186	0.0640	0.0112
Std. Dev.	3.1354	2.9784	0.9920	0.0309	1.5317	9.5841	12.1076	66.7578

Notes: The exhibit reports summary statistics for a variety of the characteristics that will be used throughout the analysis. Kurt *is the scaled fourth moment of the distribution of returns.* IdioKurt *is idiosyncratic kurtosis, which is the fourth moment of residual returns, where residuals are obtained from a daily Fama–French–Carhart four-factor model.* IdioSkew *is the idiosyncratic skewness or the skewness of residual returns.* IdioVolt *is idiosyncratic volatility or the standard deviation of daily residual returns.* Beta *is the CAPM estimate for Beta, where the dependent variable is the return for stock* i *and the independent variable is the value-weighted return. Beta, along with the idiosyncratic moments of the return distribution, are estimated during a six-month rolling window for each month* t. Size *is the market capitalization.* B/M *is the book-to-market ratio.* Illiq *is a measure of Amihud's [2002] measure of illiquidity, which is the absolute value of daily returns scaled by shares outstanding (in 100,000s).*

EXHIBIT 2
Idiosyncratic and Idiosyncratic Volatility



Note: The exhibit shows idiosyncratic kurtosis and idiosyncratic volatility for the average stock across our sample time period.

EMPIRICAL RESULTS

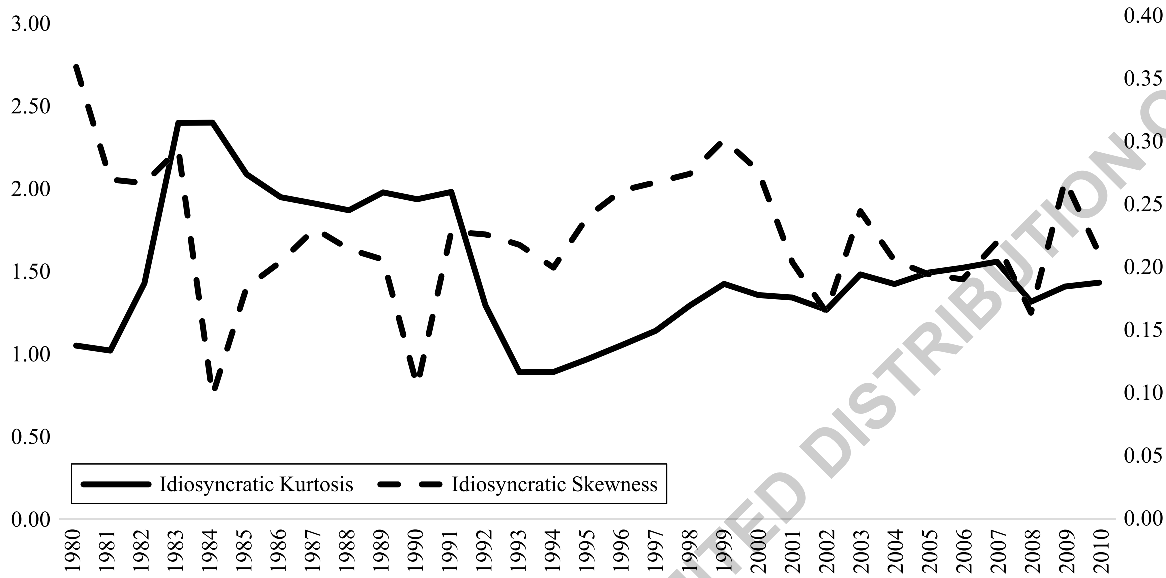
Summary Statistics and Correlation

Exhibit 1 reports statistics that describe the sample for characteristics that are used throughout the analysis. We find that the average stock in our sample has a *Kurt* of 1.5968, an *IdioKurt* of 1.5016, an *IdioSkew* of 0.2301, an *IdioVolt* of 0.0328, a *Beta* of 0.6615, and market cap (*Size*) of $1.61 billion. Further, the mean *B/M* ratio is 0.4061, while the mean *Illiq* is 1.0188.

As an additional presentation of statistics that summarize our data, we graph idiosyncratic kurtosis and idiosyncratic volatility for the average stock in our sample across time in Exhibit 2. As the figure shows, the data do not seem to be highly correlated across time. In fact, from 2002 until the end of the time period, the correlation seems to be, if anything, negative. Exhibit 3 plots idiosyncratic kurtosis and idiosyncratic skewness across time. Similar to Exhibit 2, these two measures seem to be unrelated during the sample time period.

Exhibit 3

Idiosyncratic Kurtosis and Idiosyncratic Skewness



Note: The exhibit shows idiosyncratic kurtosis and idiosyncratic skewness for the average stock across our sample time period.

Portfolio Analysis

To begin our analysis, we examine returns across value-weighted portfolios sorted by idiosyncratic kurtosis. In particular, we report a variety of next-month returns (and alphas) after forming portfolios in month t. Exhibit 4 reports the results when idiosyncratic kurtosis is measured using residual returns from a market model. For robustness, we also examined idiosyncratic kurtosis portfolios where kurtosis is obtained using residual returns from a Fama–French [1996] three-factor model and Carhart [1997] four-factor model.[2] In each case, results are similar to those reported but have not been tabulated. We report mean returns and mean excess returns in month $t + 1$. Excess returns are returns in excess of the risk-free rate, which is approximated with one-month U.S. government T-bill yields. Alphas are obtained from estimating variants of the following equation using monthly data on the five portfolios sorted by idiosyncratic kurtosis.

$$ExcessReturn_{p,t+1} = \alpha + \beta_1 MRP_{t+1} + \beta_2 SMB_{t+1} + \beta_3 HML_{t+1} + \beta_4 UMD_{t+1} + \varepsilon_{p,t+1} \quad (3)$$

The dependent variable is the excess return for portfolio p in month $t+1$. The independent variables include the market risk premium (*MRP*), the small-minus-big risk factor (*SMB*), the high-minus-low risk factor (*HML*), and the Carhart [1997] up-minus-down risk factor (*UMD*). *CAPM Alpha* is the intercept from estimating a traditional CAPM. *FF3F Alpha* is the intercept from estimating the above equation but excluding *UMD*. *FF4F Alpha* is the estimated intercept from Equation (3). We report these alphas across portfolios sorted by idiosyncratic kurtosis along with differences between extreme portfolios, with corresponding robust t-statistics in parentheses.

The first row of Exhibit 4 shows that next-month mean returns are decreasing monotonically across increasing portfolios. Column 6 shows that the difference between extreme portfolios is −0.0025 (t-statistic = −2.72). These results suggest that the return premium associated with idiosyncratic kurtosis is approximately 25 basis points (bps) per month thus indicating that the return premium is not only statistically significant, but also economically meaningful. Similar results are found in the second row of Exhibit 4. In the final three rows of Exhibit 4, we report alphas on the portfolios. We see that *CAPM Alphas* are again monotonically decreasing across increasing portfolios. The high-minus-low portfolio difference is 26 bps. Similar results are found when we examine *FF3F Alphas*

EXHIBIT 4
The Idiosyncratic Kurtosis Return Premium

	Expected Returns by Idiosyncratic Kurtosis Portfolios					
	[1] Q1	[2] Q2	[3] Q3	[4] Q4	[5] Q5	[6] Q5-Q1
Monthly Returns	0.0110	0.0104	0.0101	0.0099	0.0084	–0.0025***
						(–2.72)
Excess Returns	0.0068	0.0062	0.0060	0.0057	0.0043	–0.0025***
						(–2.72)
CAPM Alphas	0.0009	0.0002	–0.0001	–0.0004	–0.0017**	–0.0026***
	(1.62)	(0.36)	(–0.19)	(–0.66)	(–2.48)	(–2.69)
FF3F Alphas	0.0008	0.0001	–0.0002	–0.0007	–0.0023***	–0.0031***
	(1.56)	(0.12)	(–0.29)	(–1.18)	(–3.56)	(–3.54)
FF4F Alphas	0.0008	0.0003	–0.0000	–0.0006	–0.0018***	–0.0026***
	(1.33)	(0.50)	(–0.04)	(–0.82)	(–2.68)	(–2.65)

Notes: The exhibit reports various measures of next-month returns (and alphas) across portfolios sorted by idiosyncratic kurtosis in month t. Idiosyncratic kurtosis is measured using residual returns from a market model. We report mean returns and mean excess returns in month t + 1. Excess returns are returns in excess of the risk-free rate, which is approximated with one-month U.S. government T-bill yields. Alphas are obtained from estimating variants of the following equation.

$$ExcessReturn_{i,t+1} = \alpha + \beta_1 MRP_{t+1} + \beta_2 SMB_{t+1} + \beta_3 HML_{t+1} + \beta_4 UMD_{t+1} + \varepsilon_{i,t+1}$$

*The dependent variable is excess return for stock i in month t + 1. The independent variables include the market risk premium (*MRP*), the small-minus-big risk factor (*SMB*), the high-minus-low risk factor* (HML)*, and the Carhart [1997] up-minus-down risk factor (*UMD*).* CAPM Alpha *is the intercept from estimating a traditional CAPM.* FF3F Alpha *is the intercept from estimating the above equation but excluding* UMD. FF4F Alpha *is the estimated intercept from the above equation. We report these alphas across portfolios sorted by idiosyncratic kurtosis along with differences between extreme portfolios with corresponding* t*-statistics in parentheses.*

*** and *** denote statistical significance at the 5%, and 1% levels, respectively.*

and *FF4F Alphas*. The differences between extreme portfolios are −31 bps and −26 bps, respectively.

Idiosyncratic Kurtosis and the Cross Section of Returns

To better control for other factors that have been shown to influence expected returns, we next examine the cross-sectional relation between returns and idiosyncratic kurtosis. We use the Fama–MacBeth [1973] framework to estimate following equation:

$$\begin{aligned} Return_{i,t+1} = {} & \beta_0 + \beta_1 IdioKurt_{i,t} + \beta_2 IdioVolt_{i,t} \\ & + \beta_3 Beta_{i,t} + \beta_4 \ln(Size_{i,t}) + \beta_5 \ln(B/M_i) \\ & + \beta_6 Momentum_{i,t} + \beta_7 Illiq_{i,t} + \varepsilon_{i,t} \end{aligned} \quad (4)$$

The dependent variable is the return for each stock in month $t + 1$. The independent variables have been defined previously and include *IdioKurt*, *IdioVolt*, *Beta*, *ln*(*Size*), *ln*(*B/M*), *Momentum*, and *Illiq*. Here, *t*-statistics are obtained from Newey–West [1987] standard errors that include three lags. Column 1 of Exhibit 5 reports the estimates from our most basic specification that includes only *IdioKurt*. We find that the coefficient on *IdioKurt* is negative and significant. Column 2 shows that when we include both *IdioKurt* and *IdioVolt*, the estimate for *IdioKurt* remains negative and significant while the estimate for *IdioVolt* is not reliably different from zero. The negative relation between *IdioKurt* and expected returns holds in each column as we add control variables to our specifications. Column 4 of Exhibit 5 reports our most comprehensive specification. *IdioKurt* has a coefficient of −0.0369, which is significant at the 1% level. Consistent with previous research, we also find that size, value, momentum, and illiquidity are significantly related to expected returns. We do not find a significant relation between expected returns and idiosyncratic volatility or beta.

EXHIBIT 5
Fama–MacBeth [1973] Cross-Sectional Regressions

	[1]	[2]	[3]	[4]
Intercept	1.3148***	1.1777***	1.1916***	4.2379***
	(3.79)	(4.27)	(4.57)	(6.99)
$IdioKurt_{i,t}$	–0.0203*	–0.0402***	–0.0431***	–0.0369***
	(–1.93)	(–2.89)	(–3.27)	(–3.12)
$IdioVolt_{i,t}$		0.7672	1.1402	–1.7254
		(0.15)	(0.23)	(–0.44)
$Beta_{i,t}$			–0.0444	0.0068
			(–0.65)	(0.11)
$Ln(Size_{i,t})$				–0.1092***
				(–2.60)
$Ln(B/M_{i,t})$				0.6656***
				(11.14)
Momentum				0.5288***
				(3.52)
$Illiq_{i,t}$				0.1004***
				(3.99)

Notes: The exhibit reports the results from estimating the following equation using pooled stock month data using Fama–MacBeth [1973] regressions.

$$
\begin{aligned}
Return_{i,t+1} = {} & \beta_0 + \beta_1 IdioKurt_{i,t} + \beta_2 IdioVolt_{i,t} \\
& + \beta_3 Beta_{i,t} + \beta_4 \ln(Size_{i,t}) + \beta_5 \ln(B/M_{i,}) \\
& + \beta_6 Momentum_{i,t} + \beta_7 Illiq_{i,t} + \varepsilon_{i,t}
\end{aligned}
$$

The dependent variable is the return for each stock in month $t + 1$. The independent variables, which are measured for each stock in month t, include the following: IdioKurt *is the idiosyncratic kurtosis;* IdioVolt *is the idiosyncratic volatility. We note that these two idiosyncratic moments are obtained using residual returns from a daily market model. Beta* (Beta)*, the natural log of the market capitalization* (ln(Size))*, the natural log of the book-to-market ratio* (ln(B/M))*, the cumulative return from $t - 2$ to month $t - 12$* (Momentum)*, and Amihud's [2002] measure of illiquidity* (ln(Illiq))*.* t*-statistics are obtained from Newey–West [1987] standard errors that include three lags.*

** and *** denote statistical significance at the 10% and 1% levels, respectively.*

EXHIBIT 6
Fama–MacBeth [1973] Cross-Sectional Regressions

	[1]	[2]	[3]	[4]
Intercept	1.3201***	1.1714***	1.1845***	4.1998***
	(3.83)	(4.23)	(4.53)	(6.92)
$IdioKurt_{i,t}$	–0.0205**	–0.0335**	–0.0363***	–0.0317**
	(–2.15)	(–2.30)	(–2.64)	(–2.57)
$IdioSkew_{i,t}$	–0.0695	–0.1264***	–0.1273***	–0.1117***
	(–1.56)	(–4.04)	(–4.06)	(–3.82)
$IdioVolt_{i,t}$		1.3295	1.6882	–1.1098
		(0.27)	(0.34)	(–0.28)
$Beta_{i,t}$			–0.0422	0.0073
			(–0.62)	(0.12)
$Ln(Size_{i,t})$				–0.1073**
				(–2.55)
$Ln(B/M_{i,t})$				0.6632***
				(11.13)
Momentum				0.5279***
				(3.51)
$Illiq_{i,t}$				0.0997***
				(3.97)

Notes: The exhibit reports the results from estimating the following equation using pooled stock month data using Fama–MacBeth [1973] regressions.

$$
\begin{aligned}
Return_{i,t+1} = {} & \beta_0 + \beta_1 IdioKurt_{i,t} + \beta_2 IdioSkew_{i,t} \\
& + \beta_3 IdioVolt_{i,t} + \beta_4 Beta_{i,t} + \beta_5 \ln(Size_{i,t}) \\
& + \beta_6 \ln(B/M_i) + \beta_7 Momentum_{i,t} + \beta_8 Illiq_{i,t} + \varepsilon_{i,t}
\end{aligned}
$$

The dependent variable is the return for each stock in month $t + 1$. The independent variables, which are measured for each stock in month t, include the following: IdioKurt *is the idiosyncratic kurtosis.* IdioSkew *is the idiosyncratic skewness.* IdioVolt *is the idiosyncratic volatility. We note that the idiosyncratic moments are obtained using residual returns from a daily market model. Beta* (Beta)*, the natural log of the market capitalization* (ln(Size))*, the natural log of the book-to-market ratio* (ln(B/M))*, the cumulative return from $t - 2$ to month $t - 12$* (Momentum)*, and Amihud's [2002] measure of illiquidity* (ln(Illiq))*.* t*-statistics are obtained from Newey–West [1987] standard errors that include three lags.*

*** and *** denote statistical significance at the 5% and 1% levels, respectively.*

Our results are similar to previous findings that have found a somewhat puzzling negative relation between proxies for risk and expected returns (see Ang et al. [2006] and Frazzini and Pedersen [2014]). The kurtosis of stock returns seems like an intuitive measure of risk. An increased probability of ending up farther away from the mean because of kurtosis is in a similar vein to a distribution with a larger standard deviation. Thus, a negative relation between kurtosis and expected returns is either indicative of kurtosis not being a very good proxy for risk, or that nonstandard preferences somehow dominate. Thus, the next step in our analysis is to examine this relation more closely while controlling for additional distributional factors that have been shown to influence returns.

Boyer, Mitton, and Vorkink [2010] examine the relation between expected idiosyncratic skewness and expected returns and find a negative and significant relation. The rationale for this finding is most often based on a preference for lottery-like stocks. Exhibit 6 examines Fama–MacBeth cross-sectional regressions similar to those in Exhibit 5 but includes a measure of idiosyncratic skewness, which was defined previously. Given that skewness is related to the distributional characteristics of stock returns and has been shown to influence expected returns, it is an important variable to consider when examining the relation between idiosyncratic kurtosis and stock returns. Although *IdioSkew* does have a consistently negative coefficient, it does not marginally influence our findings with respect to idiosyncratic kurtosis. For instance, the estimate on *IdioKurt* is approximately 14% lower in Column 4 of Exhibit 6 than in Column 4 of Exhibit 5. However, our findings that idiosyncratic kurtosis is negatively related to expected returns does not appear to be driven by the skewness of the distribution.

CONCLUSION

This study examines the implications of kurtosis in the return distribution on asset prices in U.S. equity markets. The motivation to empirically examine how kurtosis is priced in stocks is based on two competing ideas. First, thicker tails in the return distribution might simply reflect risk that is not captured by traditional measures of volatility. Traditional asset pricing theory suggests that risk-averse investors have preferences for portfolios with lower risk. In this case, stocks with excess kurtosis might exhibit price discounts and subsequently outperform other stocks. Second, more recent theory suggests that some investors might have preferences for stocks with fatter-tailed distributions. One explanation for these preferences is based on the idea that investors with preferences for lottery-like assets might prefer stocks with larger tails. Another explanation might be that some investors face binding leverage constraints and may be willing to substitute tail risk for leverage in an attempt to realize higher returns. Regardless of which explanation has more merit, examining the implications of kurtosis on asset prices becomes an empirical question. The main objective of this study is to provide an answer to those questions.

The results of this study tend to support the second idea, as stocks with higher idiosyncratic kurtosis generally underperform stocks with lower idiosyncratic kurtosis. These results hold after controlling for a variety of factors that have already been shown to influence the cross section of stock returns—including, but not limited to, idiosyncratic volatility and idiosyncratic skewness. After holding these variables constant in a Fama–MacBeth [1973] framework, we observe a reliably negative return premium associated with idiosyncratic kurtosis.

The results from this study have important implications that suggest that while stocks typically experience nonnormal, extreme returns, some investors have a preference for stocks with these types of distributions—so much so that stocks with the most extreme returns—on either side of the distribution—exhibit contemporaneous price premiums and subsequent underperformance. Because the motives of investors are unobserved, explanations for why we observe this negative return premium may vary widely. The objective of this study is to simply document this peculiar finding and discuss its practical implications. To the extent that kurtosis reflects risk that is not accounted for in other traditional measures of volatility, investors do not appear to be compensated for such risk. Instead, our results suggest that investors generally overpay for stocks that exhibit excess kurtosis.

ENDNOTES

[1]When replicating much of the analysis that follows, we find that while the magnitude of the negative return premium changes, the level of statistical significance remains the same.

[2]Although a variety of other factors have been implemented in the Fama–French framework, we decided to limit our analysis to the most commonly used specifications.

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